# *Théorie de la synchronisation - synchronisation harmonique*

## *Oscillateur entretenu par un échappement*

### *Caractéristiques du système*

$T := 0.2 \cdot s$ $\quad \omega_0 := \dfrac{2 \cdot \pi}{T}$ $\quad J := 8 \cdot 10^{-7} \cdot kg \cdot m^2$ $\quad q_0 := 270 \cdot deg$

### Frottement visqueux

$\eta := 0.002$ $\quad C := 2 \cdot J \cdot \eta \cdot \omega_0$ $\quad F_{v\_max} := C \cdot \omega_0 \cdot q_0$ $\quad F_{v\_max} = 0.015 \, N \cdot mm$ $\quad \lambda := \dfrac{F_{v\_max}}{J \cdot \omega_0^2}$ $\quad \eta_1 := \dfrac{\eta}{\lambda}$

$\lambda = 0.019$

### Frottement constant

$f_b := 0.04$ $\quad f_1 := \dfrac{f_b}{\lambda}$ $\quad F_{c\_max} := \lambda \cdot J \cdot \omega_0^2 \cdot f_1$ $\quad F_{c\_max} = 0.032 \, N \cdot mm$

### Echappement

$\lambda_b := 48 \cdot deg$ $\quad C_e := 0.20 \cdot J \cdot \omega_0^2 \cdot q_0$ $\quad C_e = 0.744 \, N \cdot mm$ $\quad \Gamma_1 := \dfrac{C_e}{\lambda \cdot J \cdot \omega_0^2}$ $\quad E := \dfrac{\Gamma_1}{2 \cdot \pi} \cdot \lambda_b$

### Calcul du point de synchronisation

$$y_S(\eta, f, a, \varepsilon, y) := racine\left[\left(2 \cdot \eta \cdot y^2 + \frac{4}{\pi} \cdot f \cdot y - 2 \cdot E\right)^2 + \varepsilon^2 \cdot y^4 - a^2 \cdot y^2, y\right] \qquad x_S(a, \varepsilon, y) := arccos\left(\frac{\varepsilon}{a} \cdot y\right)$$

### Calcul des paramètres de stabilité

$$s(\eta, f, y) := -\eta - \frac{f}{\pi \cdot y} \qquad p(\eta, f, \varepsilon, y) := \eta^2 + \frac{2 \cdot f \cdot \eta}{\pi \cdot y} + \frac{2 \cdot f \cdot E}{\pi \cdot y^3} - \frac{E^2}{y^4} + \frac{1}{4} \cdot \varepsilon^2$$

### Amplitude du régime autonome (sans synchronisation)

$a_1 := 0$ $\quad \varepsilon := 0$ $\quad y_{auto} := \dfrac{1}{\pi \cdot \eta_1} \cdot \left(-f_1 + \sqrt{f_1^2 + \pi^2 \cdot \eta_1 \cdot E}\right)$ $\quad y_{auto} = 3.8$

### *Régimes transitoires vers un foyer attractif de grande amplitude pour $\Omega > \omega_0$*

### Excitation harmonique

$a_{seuil} := \dfrac{4 \cdot f_1}{\pi}$ $\quad a_{seuil} = 2.702$ $\quad a_1 := a_{seuil}$ $\quad F_{harm} := a_1 \cdot \left(\lambda \cdot J \cdot \omega_0^2\right)$ $\quad F_{harm} = 4.021 \times 10^{-5} \, N \cdot m$

$\varepsilon := -0.8 \cdot \dfrac{a_1}{y_{auto}}$ $\quad \varepsilon = -0.569$

$n := 500$ $\quad i := 0 .. n$ $\quad x_0 := 0$ $\quad x_1 := 2 \cdot \pi$ $\quad \Delta x := \dfrac{x_1 - x_0}{n}$ $\quad x_i := x_0 + i \cdot \Delta x$

$y := 5$ $\quad y_f := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_f := x_S(a_1, \varepsilon, y_f)$ $\quad y_f = 4.564$ $\quad x_f = 2.861$

$s(\eta_1, f_1, y_f) = -0.254$ $\quad s(\eta_1, f_1, y_f)^2 = 0.065$ $\quad p(\eta_1, f_1, \varepsilon, y_f) = 0.116$ **foyer**

$y := 2$ $\quad y_c := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_c := 2 \cdot \pi - x_S(a_1, \varepsilon, y_c)$ $\quad y_c = 2.406$ $\quad x_c = 4.181$

$s(\eta_1, f_1, y_c) = -0.387$ $\quad s(\eta_1, f_1, y_c)^2 = 0.15$ $\quad p(\eta_1, f_1, \varepsilon, y_c) = -0.528$ **col**

$b(x) := \frac{1}{\eta_1}\cdot\left(\frac{-1}{\pi}\cdot f_1 + \frac{a_1}{4}\cdot\sin(x)\right) \qquad Y_i := b(x_i) + \sqrt{b(x_i)^2 + \frac{E}{\eta_1}} \qquad X_i := \frac{a_1}{\varepsilon}\cdot\cos(x_i)\cdot\left(\frac{\pi}{2} < x_i < \frac{3\cdot\pi}{2}\right)$

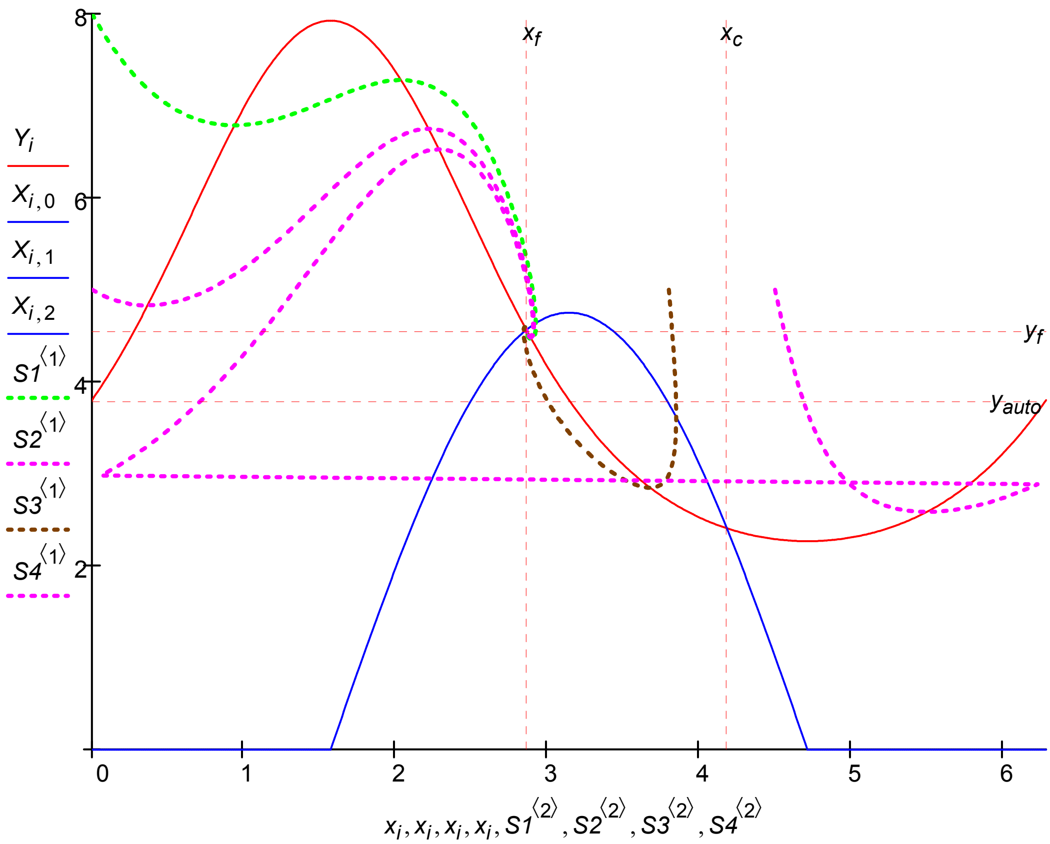
$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1\cdot Y_0 + \frac{a_1}{2}\cdot\sin(Y_1) - \frac{a_{seuil}}{2} + \frac{E}{Y_0}\right)\lambda \\ \lambda\cdot\left(\frac{-\varepsilon}{2} + \frac{a_1}{2\cdot Y_0}\cdot\cos(Y_1)\right) \end{bmatrix} \qquad t_f := 4000$$

$Y0 := \begin{pmatrix} 8 \\ 0 \end{pmatrix} \qquad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S1^{\langle 2\rangle} := mod(S1^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 5 \\ 0 \end{pmatrix} \qquad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S2^{\langle 2\rangle} := mod(S2^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 5 \\ 3.8 \end{pmatrix} \qquad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S3^{\langle 2\rangle} := mod(S3^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 5 \\ 4.5 \end{pmatrix} \qquad S4 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S4^{\langle 2\rangle} := mod(S4^{\langle 2\rangle}, 2\cdot\pi)$




## *Régimes transitoires vers un noeud attractif de grande amplitude pour $\Omega > \omega_0$*

## Excitation harmonique

$a_{seuil} := \frac{4\cdot f_1}{\pi} \qquad a_{seuil} = 2.702 \qquad \boxed{a_1 := 0.4\cdot a_{seuil}} \qquad F_{harm} := a_1\cdot\left(\lambda\cdot J\cdot\omega_0^{\,2}\right) \qquad F_{harm} = 1.608\times 10^{-5}\,N\cdot m$

$\varepsilon := -0.2\cdot\frac{a_1}{y_{auto}} \qquad \boxed{\varepsilon = -0.057}$

$y := 5 \qquad y_f := y_S(\eta_1, f_1, a_1, \varepsilon, y) \qquad x_f := x_S(a_1, \varepsilon, y_f) \qquad y_f = 4.93 \qquad x_f = 1.833$

$s(\eta_1, f_1, y_f) = -0.243 \qquad s(\eta_1, f_1, y_f)^2 = 0.059 \qquad p(\eta_1, f_1, \varepsilon, y_f) = 0.041$ **noeud**

$y := 3 \qquad y_c := y_S(\eta_1, f_1, a_1, \varepsilon, y) \qquad x_c := 2\cdot\pi - x_S(a_1, \varepsilon, y_c) \qquad y_c = 3.023 \qquad x_c = 4.553$

$s(\eta_1, f_1, y_c) = -0.33 \qquad s(\eta_1, f_1, y_c)^2 = 0.109 \qquad p(\eta_1, f_1, \varepsilon, y_c) = -0.147$ **col**

$$b(x) := \frac{1}{\eta_1}\cdot\left(\frac{-1}{\pi}\cdot f_1 + \frac{a_1}{4}\cdot sin(x)\right) \qquad Y_i := b(x_i) + \sqrt{b(x_i)^2 + \frac{E}{\eta_1}} \qquad X_i := \frac{a_1}{\varepsilon}\cdot cos(x_i)\cdot\left(\frac{\pi}{2} < x_i < \frac{3\cdot\pi}{2}\right)$$

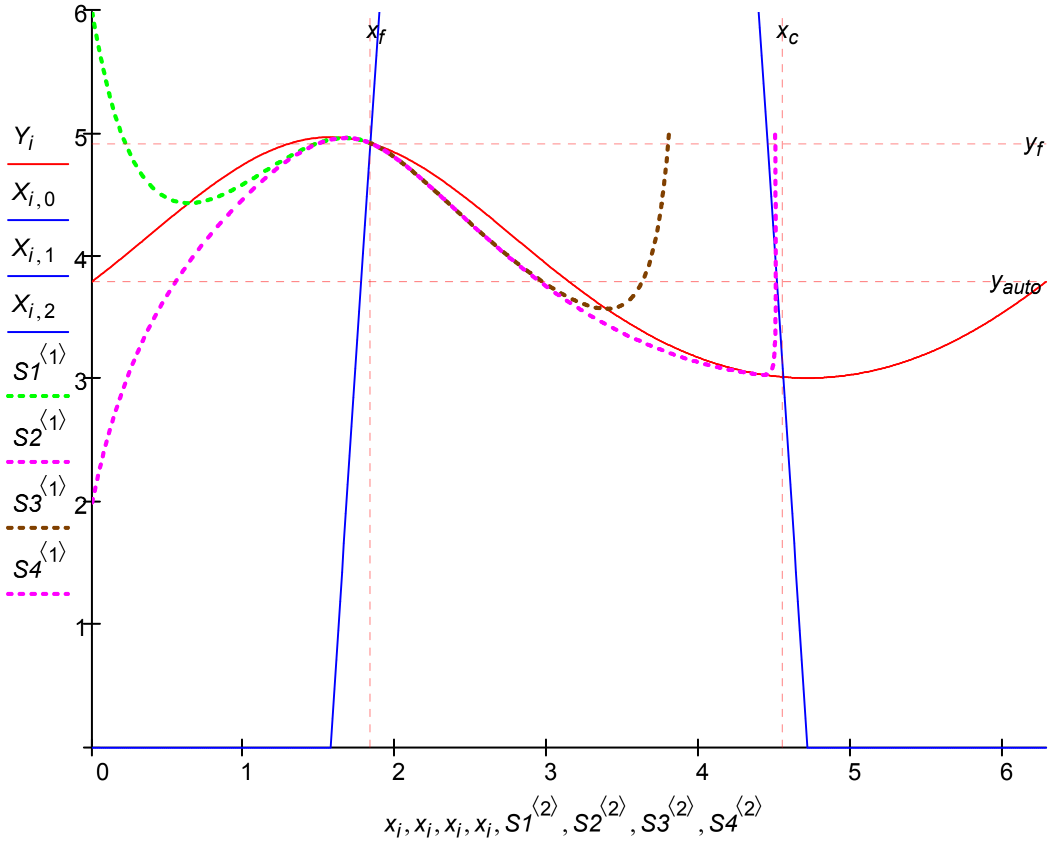
$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1\cdot Y_0 + \frac{a_1}{2}\cdot sin(Y_1) - \frac{a_{seuil}}{2} + \frac{E}{Y_0}\right)\lambda \\ \lambda\cdot\left(\frac{-\varepsilon}{2} + \frac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right) \end{bmatrix} \qquad t_f := 4000$$

$Y0 := \begin{pmatrix} 6 \\ 0 \end{pmatrix} \qquad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S1^{\langle 2\rangle} := mod(S1^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S2^{\langle 2\rangle} := mod(S2^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 5 \\ 3.8 \end{pmatrix} \qquad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S3^{\langle 2\rangle} := mod(S3^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 5 \\ 4.5 \end{pmatrix} \qquad S4 := rkfixe(Y0, 0, t_f, n, \mathbf{D}) \qquad S4^{\langle 2\rangle} := mod(S4^{\langle 2\rangle}, 2\cdot\pi)$



## Régimes transitoires vers un foyer attractif de faible amplitude pour $\Omega > \omega_0$

### Excitation harmonique

$a_{seuil} := \frac{4 \cdot f_1}{\pi}$ $\quad a_{seuil} = 2.702$ $\quad \boxed{a_1 := 1.3 \cdot a_{seuil}}$ $\quad \varepsilon := -1.2 \cdot \frac{a_1}{y_{auto}}$ $\quad \boxed{\varepsilon = -1.109}$

$y := 3$ $\quad y_f := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_f := 2\cdot\pi - x_S(a_1, \varepsilon, y_f)$ $\quad y_f = 3.012$ $\quad x_f = 3.456$

$s(\eta_1, f_1, y_f) = -0.33$ $\quad s(\eta_1, f_1, y_f)^2 = 0.109$ $\quad p(\eta_1, f_1, \varepsilon, y_f) = 0.156$ **foyer**

$y := 2$ $\quad y_c := y_S(\eta_1, f_1, a_1, \varepsilon, y)$ $\quad x_c := 2\cdot\pi - x_S(a_1, \varepsilon, y_c)$ $\quad y_c = 2.445$ $\quad x_c = 3.83$

$s(\eta_1, f_1, y_c) = -0.382$ $\quad s(\eta_1, f_1, y_c)^2 = 0.146$ $\quad p(\eta_1, f_1, \varepsilon, y_c) = -0.25$ **col**

$b(x) := \frac{1}{\eta_1}\cdot\left(\frac{-1}{\pi}\cdot f_1 + \frac{a_1}{4}\cdot \sin(x)\right)$ $\quad Y_i := b(x_i) + \sqrt{b(x_i)^2 + \frac{E}{\eta_1}}$ $\quad X_i := \frac{a_1}{\varepsilon}\cdot\cos(x_i)\cdot\left(\frac{\pi}{2} < x_i < \frac{3\cdot\pi}{2}\right)$

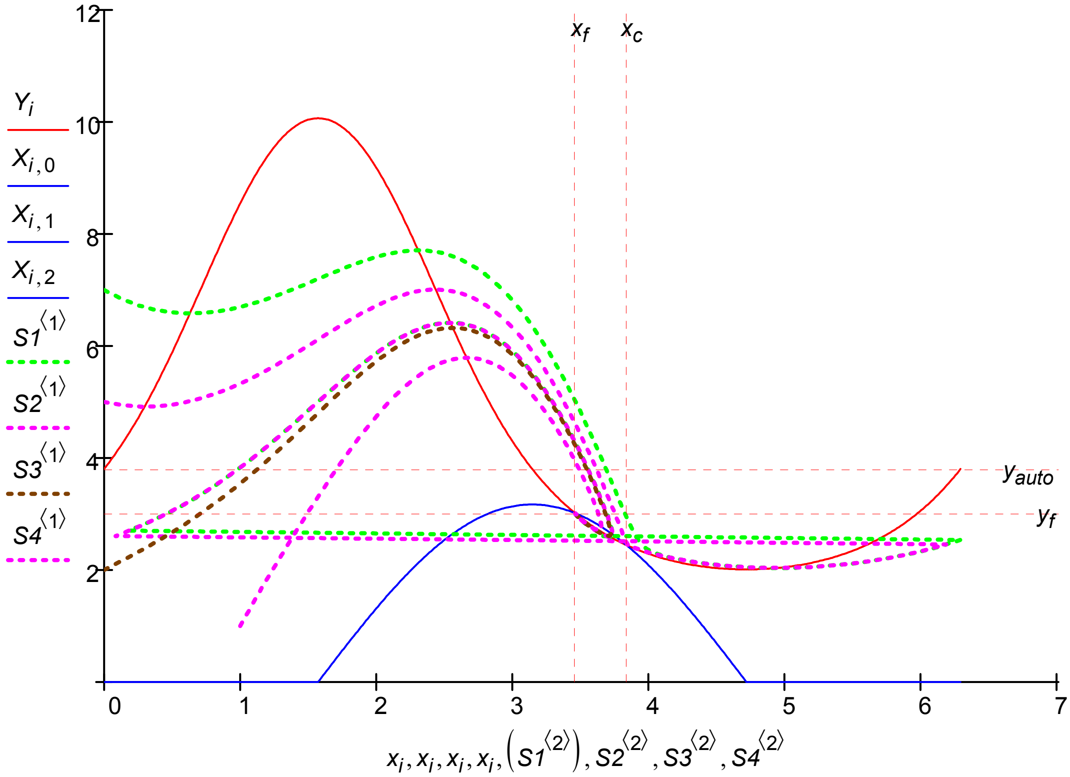
$$\mathbf{D}(t, Y) := \begin{bmatrix} \left(-\eta_1\cdot Y_0 + \frac{a_1}{2}\cdot\sin(Y_1) - \frac{a_{seuil}}{2} + \frac{E}{Y_0}\right)\lambda \\ \lambda\cdot\left(\frac{-\varepsilon}{2} + \frac{a_1}{2\cdot Y_0}\cdot\cos(Y_1)\right) \end{bmatrix} \qquad t_f := 4000$$

$Y0 := (7 \;\; 0)^T$ $\quad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S1^{\langle 2\rangle} := mod(S1^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := (5 \;\; 0)^T$ $\quad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S2^{\langle 2\rangle} := mod(S2^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := (2 \;\; 0)^T$ $\quad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S3^{\langle 2\rangle} := mod(S3^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := (1 \;\; 1)^T$ $\quad S4 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S4^{\langle 2\rangle} := mod(S4^{\langle 2\rangle}, 2\cdot\pi)$



## Limite de décrochage

$b(x) := \frac{1}{\eta_1}\cdot\left(\frac{-1}{\pi}\cdot f_1 + \frac{a_1}{4}\cdot sin(x)\right)$ $\quad Y1(x) := b(x) + \sqrt{b(x)^2 + \frac{E}{\eta_1}}$ $\quad Y2(x,\varepsilon) := \frac{a_1}{\varepsilon}\cdot cos(x)$

$z := 3.6 \qquad \varepsilon := -1$ $\quad Y1'(x) := \frac{d}{dx}Y1(x)$ $\quad Y2'(x,\varepsilon) := \frac{-a_1}{\varepsilon}\cdot sin(x)$

Soit $\quad Y1(z) = Y2(z,\varepsilon)$

$\qquad Y1'(z) = Y2'(z,\varepsilon)$

$Seuil := Trouver(z,\varepsilon) \qquad Seuil = \begin{pmatrix} 3.648 \\ -1.146 \end{pmatrix} \qquad x_{SS'} := Seuil_0 \qquad \varepsilon_1 := Seuil_1 \qquad y_{SS'} := Y1(x_{SS'})$

$\boxed{y_{SS'} = 2.681} \quad \boxed{x_{SS'} = 3.648} \quad \boxed{\varepsilon_1 = -1.146} \quad a_{seuil} := \frac{4\cdot f_1}{\pi} \quad a_{seuil} = 2.702 \quad \boxed{a_1 := 1.3\cdot a_{seuil}}$

$f_0 := \frac{\omega_0}{2\cdot\pi} \quad f_0 = 5\,Hz \quad F_{min} := \frac{f_0}{\sqrt{1-\lambda\cdot\varepsilon_1}} \quad F_{max} := \frac{f_0}{\sqrt{1+\lambda\cdot\varepsilon_1}} \quad \boxed{F_{min} = 4.947\,Hz} \quad \boxed{F_{max} = 5.055\,Hz}$

$b(x) := \frac{1}{\eta_1}\cdot\left(\frac{-1}{\pi}\cdot f_1 + \frac{a_1}{4}\cdot sin(x)\right) \quad Y_i := b(x_i) + \sqrt{b(x_i)^2 + \frac{E}{\eta_1}} \quad X_i := \frac{a_1}{\varepsilon_1}\cdot cos(x_i)\cdot\left(\frac{\pi}{2} < x_i < \frac{3\cdot\pi}{2}\right)$

$$\mathbf{D}(t,Y) := \begin{bmatrix} \left(-\eta_1\cdot Y_0 + \frac{a_1}{2}\cdot sin(Y_1) - \frac{a_{seuil}}{2} + \frac{E}{Y_0}\right)\lambda \\ \lambda\cdot\left(\frac{-\varepsilon_1}{2} + \frac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right) \end{bmatrix} \qquad t_f := 2000$$

$Y0 := \begin{pmatrix} 4 \\ 2 \end{pmatrix} \qquad S1 := rkfixe(Y0,0,t_f,n,\mathbf{D}) \qquad S1^{\langle 2\rangle} := mod(S1^{\langle 2\rangle}, 2\cdot\pi)$

$Y0 := \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad S2 := rkfixe(Y0,0,t_f,n,\mathbf{D}) \qquad S2^{\langle 2\rangle} := mod(S2^{\langle 2\rangle}, 2\cdot\pi)$